

Proof of Factorization for Exclusive Deep-Inelastic Processes ^a

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This talk summarized the proof of hard-scattering factorization for exclusive deep-inelastic processes, such as diffractive meson production.

1 Introduction

One of the interesting features of diffractive vector meson production is that it gives a novel way of probing parton densities. Donnachie and Landshoff [1] constructed a parton model for the process; their model is incorporated as an approximation in some sense in all the later work. Then Ryskin [2] showed how to estimate J/ψ production with the use of the BFKL pomeron and a constituent quark model. Brodsky et al. [3] showed how to treat the production of light vector mesons; this work was in the leading logarithm approximation in x . After that Frankfurt, Koepf, and Strikman [4] calculated the process in the leading $\ln Q^2$ approximation.

It has since been possible to prove a full factorization theorem, and it is this work [5] that is summarized here.

The proof is to all orders of perturbation theory and encompasses all logarithmic corrections, so that a systematic discussion of corrections to the leading-logarithm result is now possible. The parton densities are off-diagonal generalizations of the usual parton densities [6].

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It is interesting that the proof only applies when the virtual photon is longitudinally polarized. Transverse polarization for the photon implies power suppression, relative to the case of longitudinal polarization, and is therefore much harder to discuss with the same level of precision.

A new property, discovered while constructing the proof, is that the theorem applies to the production of all kinds of meson and at all x , in contrast to the original work [1–4], which was for vector mesons at small x . For the case of longitudinally polarized vector mesons, the parton densities are the ordinary unpolarized ones. For transversely polarized vector mesons, the parton densities are the quark transversity densities, δq , while for pseudo-scalar mesons the parton densities are the quark helicity densities, Δq . Hence the polarized parton densities can be probed in unpolarized collisions.

2 Theorem

The theorem is for the process

$$\gamma^*(q) + p \rightarrow M(q + \Delta) + p'(p - \Delta) \quad (1)$$

at large Q^2 , with t and $x = Q^2/2p \cdot q$ fixed. It asserts that the amplitude has the form

$$\sum_{i,j} \int_0^1 dz \int d\xi f_{i/p}(\xi, \xi - x; t, \mu) H_{ij}(Q^2 \xi/x, Q^2, z, \mu) \phi_j(z, \mu) \\ + \text{power-suppressed corrections}, \quad (2)$$

where f is an “off-diagonal parton density” [6], ϕ is the light-front wave function of the meson, and H is a hard-scattering coefficient, usefully computable in powers of $\alpha_s(Q)$.

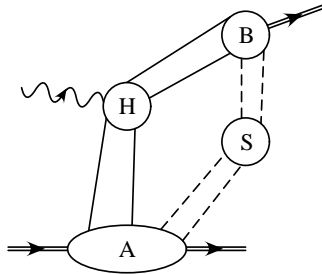


Figure 1: Typical leading region.

3 Proof

The proof follows the usual lines of a proof of factorization for an inclusive hard process [7]. We work in a frame in which the virtual photon, the proton and the meson have momentum components (in ordinary (t, x, y, z) coordinates)

$$\begin{aligned} q^\mu &= (0, 0, 0, -Q), \\ p^\mu &\approx (Q/2x, 0, 0, Q/2x), \\ V^\mu &\approx (Q/2, 0, 0, -Q/2). \end{aligned} \tag{3}$$

The approximations in the last two lines involve the neglect of masses and small transverse momenta.

Now the usual technology for obtaining the leading power behavior tells us that this comes from regions symbolized by Fig. 1, which has groups of lines that are: collinear to the target proton, collinear to the meson, and hard. In addition there may be soft lines joined to the two collinear subgraphs by gluons.

The primary difficulty is to show that the effects of the soft gluons cancel. We consider the attachments of the soft gluons to the final-state meson. After making a leading power approximation suitable to the final-state lines, a Ward identity can be applied to show that the soft attachments to the meson subgraph sum to

eikonal line connections to the lines coupling the meson to the hard subgraph. At this point, it is essential that the collinear interactions making the meson are all in the final state relative to the hard scattering. This enables a contour deformation to be made, just as in the case [8] of inclusive e^+e^- annihilation. Only after the contour deformation can the approximation be made that allows Ward identities to be used.

After that, the color singlet nature of the meson and the relative point-like nature of the hard scattering are used to show that the soft interactions cancel. (We presented this argument differently in our paper [5].) The factorization theorem Eq. (2) then follows easily.

The proof of cancellation of the soft gluon interactions is intimately related to the fact that the meson arises from a quark-antiquark pair generated by the hard scattering. Thus the pair starts as a small-size configuration and only substantially later grows to a normal hadronic size, in the meson. This implies that the parton density is a standard parton density (apart from the off-diagonal nature of its definition). For example, no rescattering corrections are needed on a nuclear target, other than those that are implicit in the definition of universal parton densities, and that would equally appear in ordinary inclusive deep-inelastic scattering. These statements all apply to the leading power.

4 Which parton densities go with which meson?

The general structure of the proof merely shows that the parton densities in Eq. (2) are any of the usual leading twist parton densities. A more detailed argument involving the spin structure is needed to show which of the parton densities is needed. Consider first the Born graphs, such as Fig. 2, for the hard scattering H .

To leading power, we ignore masses in H . Then Fig. 2 contains an odd number of Dirac matrices, which are to be contracted with

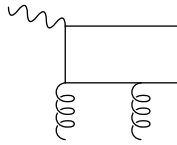


Figure 2: A typical graph for a gluon-induced hard scattering.

a matrix from the light-cone meson wave function. We have

$$\text{tr} \left[(\text{Odd \# of Dirac matrices}) \times \gamma^+ \times \begin{pmatrix} 1 \\ \gamma_5 \\ \gamma_i \end{pmatrix} \right], \quad (4)$$

where the column lists the cases for a longitudinally polarized vector meson, a pseudo-scalar meson, and a transversely polarized vector meson. (The index i is a transverse index.)

It follows immediately that we get a zero trace for the case of a transverse vector meson, which can therefore not be generated by a gluon-induced process.

In the other two cases, the trace is zero unless the photon is longitudinal. Moreover, charge-conjugation invariance kills the case of the pseudo-scalar meson. Thus we find that the gluon-induced subprocess is associated only with the unpolarized gluon density and only with longitudinal vector meson production from a longitudinal photon. All other cases are power suppressed.

Similar arguments for quark-induced processes give the same results, except that transverse vector meson production is associated with the transversity density δq and pseudo-scalar meson production with the helicity density Δq .

The arguments generalize to all orders of perturbation theory in the hard scattering.

There is one complication: A transversely polarized vector meson can result from a hard scattering with just two quark lines instead of four. The soft subgraph then has two external quark

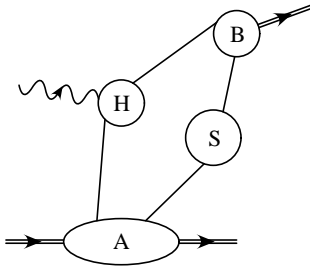


Figure 3: This region is leading for a transversely polarized vector meson, provided that there is a sufficiently soft quark (with possible gluons).

lines (plus optional additional gluons) – Fig. 3. We gain a power of Q because the hard scattering has fewer external lines, but lose a power because of the soft quarks. This contribution is called an endpoint contribution, because it probes the meson’s wave function at $z \rightarrow 0$ (or 1). In the case of longitudinal polarization, the endpoint term is suppressed relative to the non-endpoint term, but for transverse polarization, both contributions are comparable until one appeals to a Sudakov suppression. This means that it is substantially harder to give a quantitative discussion of transverse polarization than is the case for longitudinal polarization. The most important fact, however, is that both contributions for transverse polarization are a power of Q smaller than the amplitude for longitudinal polarization.

5 Pseudo-scalar meson production

Exclusive pion production involves the helicity parton densities. So it should not be suppressed at large x compared to vector meson production. But it should be much smaller at small x .

A number of predictions [5] can be made for ratios of cross sections of different mesons, if some approximations are made. These are that the meson wave functions are $SU(3)$ symmetric, that the

strange quark helicity density Δs is small, and that the helicity distribution of the up and down quarks are approximately equal and opposite: $\Delta d \approx -\Delta u$ (this follows from the observation that F_2 for the deuteron is small and the assumption that this same property is valid for the off-diagonal parton densities). We therefore predict that

$$\begin{aligned}\frac{d\sigma(e+p \rightarrow \eta+p)/dt}{d\sigma(e+p \rightarrow \pi^0+p)/dt} &\approx \frac{1}{3} \left(\frac{2\Delta u_V - \Delta d_V}{2\Delta u_V + \Delta d_V} \right)^2 \approx 3, \\ \frac{d\sigma(e+p \rightarrow \eta+p)/dt}{d\sigma(e+n \rightarrow \eta+n)/dt} &\approx \left(\frac{2\Delta u_V - \Delta d_V}{2\Delta d_V - \Delta u_V} \right)^2 \approx 1, \\ \frac{d\sigma(e+p \rightarrow \pi^0+p)/dt}{d\sigma(e+n \rightarrow \pi^0+n)/dt} &\approx \left(\frac{2\Delta u_V + \Delta d_V}{2\Delta d_V + \Delta u_V} \right)^2 \approx 1.\end{aligned}\quad (5)$$

Here $\Delta u_V = \Delta u - \Delta \bar{u}$ and $\Delta d_V = \Delta d - \Delta \bar{d}$.

6 Conclusions

We have a full proof of hard-scattering factorization not only for exclusive vector meson production in DIS, but also for processes such as $\gamma^* + p \rightarrow \pi^+ + n$. Among the new results are that: (a) the theorem holds for large x as well as small x , (b) it works for any meson, (c) some polarized parton densities, including the elusive transversity density, can be probed in unpolarized collisions, and (d) there is a power suppression of transverse vector meson production, beyond leading-logarithm approximation, but only at small x , because of the properties of polarized parton densities.

Acknowledgments

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1. A. Donnachie and P.V. Landshoff, *Phys. Lett. B* **185**, 403 (1987); *Nucl. Phys. B* **311**, 509 (1989).
2. M.G. Ryskin, *Z. Phys. C* **57**, 89 (1993).
3. S.J. Brodsky, L. Frankfurt, J.F. Gunion, A.H. Mueller, and M. Strikman, *Phys. Rev. D* **50**, 3134 (1994).
4. L. Frankfurt, W. Koepf, and M. Strikman, *Phys. Rev. D* **54**, 3194 (1996).
5. J.C. Collins, L. Frankfurt, and M. Strikman, *Phys. Rev. D* **56**, 2982 (1997).
6. J. Bartels and M. Loewe, *Z. Phys. C* **12**, 263 (1982);
B. Geyer *et al.*, *Z. Phys. C* **26**, 591 (1985);
T. Braunschweig *et al.*, *Z. Phys. C* **33**, 275 (1987);
I.I. Balitsky and V.M. Braun, *Nucl. Phys. B* **311**, 541 (1989);
A.V. Radyushkin, *Phys. Lett. B* **380**, 417 (1996);
X.-D. Ji, *Phys. Rev. D* **55**, 7114 (1997).
7. J.C. Collins, D.E. Soper, and G. Sterman, *Nucl. Phys. B* **261**, 104 (1985) and **B308**, 833 (1988);
G. Bodwin, *Phys. Rev. D* **31**, 2616 (1985); **34**, 3932 (1986).
8. J.C. Collins and G. Sterman, *Nucl. Phys. B* **185**, 172 (1981).